The University of Texas at Tyler

Houston Engineering Center

MENG 3303

Fall 2019 Design Project

Design Project Final Report

Professor: Aws Al-shalash

Group 1: Abdul Gaddi, Alan Nguyen, Amin Marashi, Anishkumar Patel
Table of Content

1. Table of Content ................................................................. 1
2. Introduction ........................................................................... 2
3. Problem Statement ............................................................... 3
4. Objectives ............................................................................. 4
5. Preliminary Literature Review .............................................. 5
6. Design Process ...................................................................... 6
7. Discussion ............................................................................. 9
8. Conclusion ........................................................................... 16
9. References ........................................................................... 17
10. Appendix ............................................................................ 18
**Introduction:**

The goal of this project is to familiarize students with mechanical mechanisms and designing those mechanisms in SolidWorks. Nowadays, mechanisms have become complicated in manufacturing industries. There are many mechanisms that have a complex design and are highly expensive, so it is difficult to find a cost-efficient and simple design. The use of the mechanism is important in industries. However, because of the advancement of technology, it is hard to find a simple mechanism that provides all mechanical advantages. The six-bar press the mechanism is a multifunctional, highly precise, simple, and highly cost-effective mechanism with the functionality of press. The six-bar linkage press can perform multiple operations such as punching and forging. The mechanism has a simple design, so people can manufacture the press. Moreover, a six-bar press is very precise for a continuous operation and it does not require skilled labor.
**Problem Statement:**

The purpose of this project is to design a mechanism that is multifunctional, economical, highly precise, and cost-efficient. This mechanism should have a specific functionality which is pressing and applying force. Because of the cutting-edge competition in the industries all mechanisms have become economical and efficient. The goal is to design a mechanism that is simple, strong and durable. In order to create the design, each group has to make sure that their design fits all the requirements. In order to create this mechanism, the steps of creating the mechanism need to be identified. The position synthesis, velocity synthesis, acceleration synthesis, and force analysis have to be performed. The functionality and the purpose for this design is the most important thing to be considered. The function of pressing should be seamless and smooth.

In order to come up with this mechanism each group had to consider some design requirements. For starters, this mechanism should perform a specific task that has a real-world application. The mechanism should be a six-bar linkage or higher, this mechanism should be powered by a crank or a dyad crank, the prototype for this mechanism should not exceed 1.5 ft by 1.5 ft frame, and the final requirement is that each group must reference at least two peer-reviewed journal papers that helped them design their mechanism.
Objectives:

For this design, group 1 designs a six-bar that presses. The functionality of this design is highly important. The material used for this design should be cost efficient and reliable. The press should be effective, and it should be able to apply enough force during each rotation. The six-bar press mechanism has 6 links, one input function which can be crank, and gear or cam. This design is a Grashof which means that at least one of the links makes a full rotation.

One of the considerations for this mechanism design is the material that will be used in manufacturing process. The design requires the material that is able to bear high pressure and reaction forces.

In order to do this project, certain steps must be followed. The first step drawing the diagram and coming us with the dimensions. After that, each team needs to calculate the position, velocity, acceleration, and forces. For calculating the position and velocity, the mechanism is divided into two four-bars for easier calculation. This method is done by using two vector loops. The next step would be to design the mechanism by utilizing SolidWorks. The final step is to 3D print the mechanism.
Preliminary Literature Review:

Servo press plays an important role in the six-bar linkage mechanism. In this design, press is the fifth link which is the output of the design. Dwelling will play a big part since it will make the structure stable by reducing the speed of the mechanism. The press will maintain the structure moving by adjusting speed and position of the mechanism and will hold the weight of the design so it does not affect the ground link of the structure with greater force. It will maintain the weight of the structure by absorbing energy generated from other links. One of the articles relates to the project in a sense that for their input function the input is PMSM which is Permanent Magnet Synchronous Motor which has been replaced by traditional crank input in our design process. So the crank will create the constant desired speed depending on the parameters designed in the project, which will create the rotationality of the links. Crank will be second link which is the input in the design and will serve as the function of adjusting velocity, acceleration of the links and rotationality at the same time.
Design Process I: Positions Analysis

The use of an application like SolidWorks is recommended in order to design this mechanism. SolidWorks allows users to design each link separately and connect them all together at the end. Students can see how every link will effectively work with other links. Students can also test different materials to see how their mechanisms will perform under different forces. SolidWorks is a great tool that allows students to be creative and try different things. The process begins with a position diagram on paper, then analyze positions by vector loop methods, and hand calculate velocity and acceleration of the crank in the mechanism. Create a motion study on SolidWorks to predict on position theory, velocity, and acceleration. Next, export the data from the diagram to Mathcad to verify that the analytical models are correct. After the calculation is completed, input all of the dimensions into Linkage Design Software to make sure the mechanism is work as expected velocity.

Figure 1. 2D Mechanism Designed by Linkage Design Software
Design Process II: 3D Model

The process will start from the base of the mechanism, the base will include all of the fixed points. First, the thickness selected for the mechanism based on the calculation. Second, the diameter of each of the joint is calculated and designed. In this case, the diameter will be the same for all of the joints. Finally, locate all of the pole for all of the fixed points. Sketch the base model with input dimensions. Design link two, link three and link five. The design process will be the same for all of these links. The only difference is the length of each of the link. Link four is a ternary link. Link six is a ternary link. One of the side of this link is the output. In this case, the output point is the hammer.

Figure 2. Final Assembly Mechanism in SolidWorks
Design Process II: 3D Model

**Note:** The SolidWorks model will be scaled down and used to build the mechanism. In this case, the SI unit system had been used. The scale is based on the prototype measurements.

Figure 3. ANSI Mechanical Layout of The Mechanism
**Discussion: Position Analysis**

The vector loop methods was used to derive equations for the output angle. In order to do the position, velocity, and acceleration analysis, the six-bar design needs to be broken down to two four-bars. That will make the calculation much easier. The first four-bar is positioned horizontally with reference to global xy axis in a way that would make theta 1 equal to zero. The input for the four-bar, is all the links and the angle theta 2. From there, theta 3, and theta 4 are calculated. The position of the links will depend on those angles. In this case, theta 2 is a 45-degree angle. The output calculated from the first four-bar, will be the input for the second four-bar. The position equations for the first four-bar:

\[
K_1 := \frac{f}{a} \quad K_2 := \frac{b}{g} \quad K_3 := \frac{a^2 - b^2 + g^2 + f^2}{2 a \cdot g} \quad K_4 := \frac{f}{b} \quad K_5 := \frac{g^2 - f^2 - a^2 - b^2}{2 a \cdot b}
\]

\[
A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \sin(\theta_2)
\]

\[
C := K_1 - (K_2 + 1) \cos(\theta_2) + K_3 \quad D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5
\]

\[
E := -2 \sin(\theta_2) \quad F := K_1 - (K_4 + 1) \cos(\theta_2) + K_5
\]

\[
\theta_{41} := 2 \tan^{-1} \left( \frac{-B + \sqrt{B^2 - 4 A \cdot C}}{2 \cdot A} \right) \quad \theta_{42} := 2 \tan^{-1} \left( \frac{-B - \sqrt{B^2 - 4 A \cdot C}}{2 \cdot A} \right)
\]

\[
\theta_{31} := 2 \tan^{-1} \left( \frac{-E + \sqrt{E^2 - 4 D \cdot F}}{2 \cdot D} \right) \quad \theta_{32} := 2 \tan^{-1} \left( \frac{-E - \sqrt{E^2 - 4 D \cdot F}}{2 \cdot D} \right)
\]
Discussion: Position Analysis

The position equations for the second four-bar:

\[ G = \cos(\theta_4) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad H = -2 \sin(\theta_4) \]

\[ I = K_1 - (K_2 + 1) \cos(\theta_4) + K_3 \quad J = \cos(\theta_4) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \]

\[ K = -2 \sin(\theta_4) \quad L = K_1 - (K_4 + 1) \cos(\theta_4) + K_5 \]

\[ \theta_4_1 = 2 \tan \left( \frac{-H + \sqrt{H^2 - 4 \cdot G \cdot I}}{2 \cdot G} \right) \quad \theta_4_1 = 2 \tan \left( \frac{-H - \sqrt{H^2 - 4 \cdot G \cdot I}}{2 \cdot G} \right) \]

\[ \theta_3_1 = 2 \tan \left( \frac{-K + \sqrt{K^2 - 4 \cdot J \cdot L}}{2 \cdot J} \right) \quad \theta_3_1 = 2 \tan \left( \frac{-K - \sqrt{K^2 - 4 \cdot J \cdot L}}{2 \cdot J} \right) \]
Discussion: Velocity & Acceleration Analysis

After doing the position analysis, the velocity analysis is performed. In order to get the velocity of the links, the derivative of the position needs to be calculated. This process is simplified since theta 1 equals to zero, the link ‘f’ or R6 would be constant and the derivative of any constant is zero. After obtaining the results for velocity, the derivative of velocity is calculated which is the acceleration. The calculations for the second four-bar is performed the same way. A constant angular velocity input is set in 15 RPM and that converted to 1.571 Rad/sec. Velocity analysis equations for the first four-bar:

\[
\begin{align*}
\omega_3 &= \frac{a \cdot \omega_2}{b} \cdot \frac{\sin (\theta_4 - \theta_3)}{\sin (\theta_3 - \theta_4)} \\
\omega_4 &= \frac{a \cdot \omega_2}{g} \cdot \frac{\sin (\theta_2 - \theta_3)}{\sin (\theta_4 - \theta_3)} \\
\omega_5 &= \frac{a \cdot \omega_2}{b} \cdot \frac{\sin (\theta_4 - \theta_3)}{\sin (\theta_3 - \theta_4)} \\
\omega_6 &= \frac{a \cdot \omega_2}{g} \cdot \frac{\sin (\theta_2 - \theta_3)}{\sin (\theta_4 - \theta_3)}
\end{align*}
\]

Figure 6. Constant Velocity at Input Position

Figure 3 shows the constant angular velocity of R6 link at 1.571 Rad/sec which is attached to the ground surface in first four bar operation.
\[ V_{Ax} = -a \cdot \omega^2 \cdot \sin(\theta_2) \quad V_{Ay} = -a \cdot \omega^2 \cdot \cos(\theta_2) \quad V_A = \sqrt{V_{Ax}^2 + V_{Ay}^2} \]

The acceleration is calculated by the first derivative of velocity equations. The purpose is to have an acceleration complete the press cycle in every 2 seconds. The acceleration analysis equations based the input velocity.

\[ A := g \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := g \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3) \]

\[ C := 0 + a \cdot \omega^2 \cdot \cos(\theta_2) + B \cdot \omega^3 \cdot \cos(\theta_3) - g \cdot \omega^2 \cdot \cos(\theta_4) \]

\[ F := 0 + a \cdot \omega^2 \cdot \sin(\theta_2) + B \cdot \omega^3 \cdot \sin(\theta_3) - g \cdot \omega^2 \cdot \sin(\theta_4) \]

\[ \alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \]

Figure 7.

Output Angular Velocity

Figure 5 shows the output angular velocity of link AB or R2 and 04B or R7 link of the mechanism. These angular velocities will change depending upon the input angular velocity of link 02A or R1 from our desired choice.
Figure 6 shows the output angular acceleration of link AB or R2 and 04B or R7 link of the mechanism. These outputs angular acceleration comes from the derivation of velocity with respect to time of the respective links. So depending upon the angular velocities of the mechanism, acceleration will change directly in response of those velocities.

Forces and the moment of inertia is analysis by applied external force or torque. In this case, the whole mechanism is fixed by mounted to the ground. Therefore, the only torque is applied at the motor of the crank and at all of the connection joins.

Based on vector loop methods, the force in each position of the link is analysis by these equations:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-R_{12y} & R_{12x} & 1
\end{bmatrix}
\begin{bmatrix}
F_{12x} \\
F_{12y} \\
T_{12}
\end{bmatrix} =
\begin{bmatrix}
m_1 \cdot a_{CMx} - F_{12x} \\
m_1 \cdot a_{CMy} - F_{12y} \\
I_{CM} \cdot \alpha - (R_{12x} \cdot F_{12y} - R_{12y} \cdot F_{12x})
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-R_{16y} & R_{16x} & 1
\end{bmatrix}
\begin{bmatrix}
F_{16x} \\
F_{16y} \\
T_{16}
\end{bmatrix} =
\begin{bmatrix}
m_6 \cdot a_{CMx} - F_{16x} \\
m_6 \cdot a_{CMy} - F_{16y} \\
I_{CM} \cdot \alpha - (R_{16x} \cdot F_{16y} - R_{16y} \cdot F_{16x})
\end{bmatrix}
\]
Applied motion study in SolidWorks Simulation for torque analysis. The material specified in stainless steel (SS316). The weight was determined based on density of material and dimensions of the mechanism. In this case, the torque applied was specified by external load, radius of the crank and the output angle. Based on the motion study from SolidWorks, the stainless steel is working fine for the mechanism with 100 N.m of torque and counterclockwise direction. The deformation rate is extremely low, and the increasing thickness of the area around fixed points is necessary to keep the strength of the whole system. Based on the data above the output force on the y direction is around 36.967 pounds of force.
Figure 9. Torque Applied with Plastic Properties in SolidWorks Simulations

Figure 10. Torque Applied with Stainless Steel 316 in SolidWorks Simulations
Conclusion/Concluding Remarks:

This mechanism is highly effective because it meets all the requirements of crank, cam and six-bar linkage. The cost of the mechanism will be efficient since all the materials used in the mechanism have low prices in the marketplace. This mechanism is different from other mechanisms because all output functions of cam and link bars depend on the crank function, which is our input; all other functions totally rely on the crank. Since the crank is the input generator, whichever direction the crank will rotate, all other functions in the mechanism will respond according to that. Therefore, the direction of the whole mechanism can be changed by changing crank direction of rotation from clockwise to counterclockwise or vice versa. After researching other labs related to this mechanism, team members realized that they could have made their mechanisms more advanced by adding some other function generators in between one of the links or to the cam attachment surface, which could have given new dynamics to the mechanism.
References:


Appendix: Project Gantt Chart

MENG 3303 - Professor: Aws Al-shalash

Table 1. Design Project Planner

Note: The model will be manufactured by 3D printing process with the main material is plastic. The manufacturing process will take place at Houston West Institution building. The final process is to test the plastic model and record the results. The final report will be processed based on the performance results.
Appendix: Calculations

First four-bar position

\[ R_1 + R_2 - R_7 - R_6 = 0 \]

\[ e^{j\theta_2} + e^{j\theta_3} - g e^{j\theta_4} - f e^{j\theta_1} = 0 \]

\[ a \cos \theta_2 + b \cos \theta_3 - g \cos \theta_4 - f = 0 \]

\[ a \sin \theta_2 + b \sin \theta_3 - g \sin \theta_4 = 0 \]

\[ \begin{align*}
    b \cos \theta_3 &= -a \cos \theta_2 + g \cos \theta_4 + f \\
    b \sin \theta_3 &= -a \sin \theta_2 + g \sin \theta_4
\end{align*} \]

\[ b^2 (\cos^2 \theta_3 + \sin^2 \theta_3) = (-a \cos \theta_2 + g \cos \theta_4 + f)^2 + (-a \sin \theta_2 + g \sin \theta_4)^2 \]

\[ b^2 = (-a \cos \theta_2 + g \cos \theta_4 + f)^2 + (-a \sin \theta_2 + g \sin \theta_4)^2 \]
\[
\theta_4 = 2 \tan^{-1} \left( \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right)
\]

\[
A = \cos \theta_1 - k_1 - k_2 \cos \theta_2 + k_3
\]

\[
B = -2 \sin \theta_1
\]

\[
C = k_1 - (k_2 + 1) \cos \theta_2 + k_3
\]

\[
k_1 = \frac{f}{a} \quad k_2 = \frac{b}{g} \quad k_3 = \frac{a^2 - b^2 + g^2 + f^2}{2ag}
\]

\[
\begin{align*}
g \cos \theta_4 &= a \cos \theta_2 + b \cos \theta_3 - f \\
g \sin \theta_4 &= a \sin \theta_2 + b \sin \theta_3
\end{align*}
\]

\[
g^2 \left( \sin^2 \theta_4 + \cos^2 \theta_4 \right) = \left( a \cos \theta_2 + b \cos \theta_3 - f \right)^2 + \left( a \sin \theta_2 + b \sin \theta_3 \right)^2
\]

\[
g^2 = \left( a \cos \theta_2 + b \cos \theta_3 - f \right)^2 + \left( a \sin \theta_2 + b \sin \theta_3 \right)^2
\]

\[
\theta_3 = 2 \tan^{-1} \left( \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right)
\]

\[
D = \cos \theta_2 - k_4 \cos \theta_2 + k_5
\]

\[
E = -2 \sin \theta_2
\]

\[
F = k_1 + (k_2 - 1) \cos \theta_2 + k_5
\]

\[
k_4 = \frac{f}{b} \quad k_5 = \frac{g^2 - f^2 - a^2 - b^2}{2ab}
\]
Velocity

\[ a e^{i\theta_2} + b e^{i\theta_3} - g e^{i\theta_4} = f = 0 \]
\[ j a e^{i\theta_2} \frac{d\theta_2}{dt} + j b e^{i\theta_3} \frac{d\theta_3}{dt} - j g e^{i\theta_4} \frac{d\theta_4}{dt} = 0 \]
\[ j a e^{i\theta_2} w_2 + j b e^{i\theta_3} w_3 - j g e^{i\theta_4} w_4 = 0 \]
\[ j a w_2 (\cos \theta_2 + j \sin \theta_2) + j b w_3 (\cos \theta_3 + j \sin \theta_3) - j g w_4 (\cos \theta_4 + j \sin \theta_4) = 0 \]
\[ j a w_2 \cos \theta_2 - a w_2 \sin \theta_2 + j b w_3 \cos \theta_3 - b w_3 \sin \theta_3 - j g w_4 \cos \theta_4 + g w_4 \sin \theta_4 = 0 \]
\[ \text{Re: } -a w_2 \sin \theta_2 - b w_3 \sin \theta_3 + g w_4 \sin \theta_4 = 0 \]
\[ \text{Im: } a w_2 \cos \theta_2 + b w_3 \cos \theta_3 - g w_4 \cos \theta_4 = 0 \]

\[ \times w_3 = \frac{a w_2}{b} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \]
\[ \times w_4 = \frac{a w_2}{g} \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \]

\[ \times v_A = j a e^{i\theta_2} w_2 = a w_2 (j \cos \theta_2 - \sin \theta_2) \]
\[ v_{BA} = j b e^{i\theta_3} w_3 = b w_3 (j \cos \theta_3 - \sin \theta_3) \]
\[ v_B = j g e^{i\theta_4} w_4 = g w_4 (j \cos \theta_4 - \sin \theta_4) \]
\[ v_{Ax} = -a w_2 \sin \theta_2, \quad v_{Ay} = a w_2 \cos \theta_2 \]
\[ \|v_A\| = \sqrt{v_{Ax}^2 + v_{Ay}^2} \quad \theta = \tan^{-1} \left( \frac{v_{Ay}}{v_{Ax}} \right) \]
Acceleration
\[ j \alpha_2 e^{j \theta_2} + j b w_3 e^{j \theta_3} - j g w_4 e^{j \theta_4} = 0 \]
\[ j a \frac{dw_2}{dt} e^{j \theta_2} - a w_2 e^{j \theta_2} \frac{d \theta_2}{dt} + j b \frac{dw_2}{dt} e^{j \theta_3} - b w_3 e^{j \theta_3} \frac{d \theta_3}{dt} - j g a_4 e^{j \theta_4} + g w_4 e^{j \theta_4} \frac{d \theta_4}{dt} = 0 \]
\[ j a_2 \left( \cos \theta_2 + j \sin \theta_2 \right) - a w_2^2 \left( \cos \theta_2 + j \sin \theta_2 \right) + j b a_3 \left( \cos \theta_3 + j \sin \theta_3 \right) - b w_3^2 \left( \cos \theta_3 + j \sin \theta_3 \right) - j g a_4 \left( \cos \theta_4 + j \sin \theta_4 \right) + g w_4^2 \left( \cos \theta_4 + j \sin \theta_4 \right) = 0 \]
\[ j a_2 \cos \theta_2 - a_2 \sin \theta_2 - a w_2^2 \cos \theta_2 - a w_2^2 j \sin \theta_2 + j b a_3 \cos \theta_3 - b a_3 \sin \theta_3 - b w_3^2 \cos \theta_3 - j b w_3^2 \sin \theta_3 - j g a_4 \cos \theta_4 + g a_4 \sin \theta_4 + g w_4^2 \cos \theta_4 + j g w_4^2 \sin \theta_4 = 0 \]
Re: \[ a_2 \sin \theta_2 - a w_2^2 \cos \theta_2 + b a_3 \sin \theta_3 - b w_3^2 \cos \theta_3 - g a_4 \sin \theta_4 + g w_4^2 \sin \theta_4 \]
Im: \[ -a_2 \cos \theta_2 - a w_2^2 \sin \theta_2 + b a_3 \cos \theta_3 - b w_3^2 \sin \theta_3 + g a_4 \cos \theta_4 + g w_4^2 \sin \theta_4 \]
\[ \alpha_3 = \frac{CD - AF}{AE - BD} \]
\[ \alpha_4 = \frac{CE - BF}{AE - BD} \]
\[ A = g \sin \theta_4 \quad B = b \sin \theta_3 \]
\[ C = a a_2 \sin \theta_2 + a w_2^2 \cos \theta_2 + b w_3^2 \cos \theta_3 - g w_4^2 \cos \theta_4 \]
\[ D = g \cos \theta_4 \quad F = a a_2 \cos \theta_2 - a w_2^2 \sin \theta_2 - b w_3^2 \sin \theta_3 + g w_4^2 \sin \theta_4 \]
\[ E = b \cos \theta_3 \]
\[ A_A = a a_2 \left( -\sin \theta_2 + j \cos \theta_2 \right) - a w_2^2 \left( \cos \theta_2 + j \sin \theta_2 \right) \]
\[ A_B = b a_3 \left( -\sin \theta_3 + j \cos \theta_3 \right) - b w_3^2 \left( \cos \theta_3 + j \sin \theta_3 \right) \]
\[ A_B = g a_4 \left( -\sin \theta_4 + j \cos \theta_4 \right) - g w_4^2 \left( \cos \theta_4 + j \sin \theta_4 \right) \]
Force Analysis First four-bar

\[ \Sigma F = m_a \]
\[ \Sigma T = l_{CM} \alpha \]

\[ \Sigma F_x = F_{16} + F_{12} \cos \theta_3 = m_1 \alpha_{CMX} \]
\[ \Sigma F_y = m_1 \alpha_{CMY} \]

\[ \Sigma T = T_{12} + ( R_{12} \times F_{12} ) + ( R_{16} \times F_{16} ) = l_{CM} \alpha \]

\[ R_{12} \times F_{12} = \begin{vmatrix} R_{12x} & R_{12y} \\ R_{12x} & R_{12y} \end{vmatrix} \]

\[ F_{12} = F_{12x} + F_{12y} = F_{12} \cos \theta_3 + F_{12} \sin \theta_3 \]

\[ F_{12x} = m_1 \alpha_{CMX} \]

\[ F_{12y} = m_1 \alpha_{CMY} \]

\[ T_{12} + ( R_{12x} F_{12y} - R_{12y} F_{12x} ) + ( R_{16x} F_{16y} - R_{16y} F_{16x} ) = l_{CM} \alpha \]

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12y} & R_{12x} & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_1 \alpha_{CMX} - F_{16x} \\ m_1 \alpha_{CMY} - F_{16y} \\ l_{CM} - ( R_{16x} F_{16y} - R_{16y} F_{16x} ) \end{bmatrix} \]

\[ |F_{12}| = \sqrt{F_{12x}^2 + F_{12y}^2} \]

\[ F_{12} = 180 + \tan^{-1} \left( \frac{F_{12y}}{F_{12x}} \right) \]

\[ \Sigma F_x = -F_{12x} + F_{23x} = m_2 \alpha_{CMX} \]
\[ \Sigma F_y = -F_{12y} + F_{23y} + F_3 = m_2 \alpha_{CMY} \]

\[ \Sigma T = T_{12} + T_{23} + ( R_{12} \times F_{12} ) + ( R_{23} \times F_{23} ) = l_{CM} \alpha \]

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{23y} R_{12x} & 1 \end{bmatrix} \begin{bmatrix} F_{23x} \\ F_{23y} \\ T_{23} \end{bmatrix} = \begin{bmatrix} m_2 \alpha_{CMX} + F_{12x} + F_{23} \\ m_2 \alpha_{CMY} - F_{23y} + F_3 \\ l_{CM} - ( R_{23x} F_{23y} - R_{23y} F_{23x} ) \end{bmatrix} \]
\[ \Sigma F = ma \]
\[ \Sigma T = I_{cm} \alpha \]
\[ \Sigma F_x = F_{16x} + F_6 = m_6 a_{cmx} \]
\[ \Sigma F_y = F_{16y} = m_6 a_{cmy} \]
\[ \Sigma T = T_{16} + (R_{16x} F_{16}) + (R_6 x F_6) = I_{cm} \alpha \]

\[ R_{16x} F_{16} = R_{16x} F_{16y} - R_{16y} F_{16x} = \begin{bmatrix} R_{16x} & R_{16y} \\ F_{16x} & F_{16y} \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ R_{16y} & R_{16x} \end{bmatrix} \begin{bmatrix} F_{16x} \\ F_{16y} \\ T_{16} \end{bmatrix} = \begin{bmatrix} m_6 a_{cmx} - F_{16x} \\ m_6 a_{cmy} - F_{16y} \\ I_{cm} \alpha - (R_{16y} F_{16x} - R_{16x} F_{16y}) \end{bmatrix} \]

\[ F_{16} = \sqrt{F_{16x}^2 + F_{16y}^2} \quad F_{16} = 180 + \tan^{-1} \left( \frac{F_{16y}}{F_{16x}} \right) \]

\[ \Sigma F_x = -F_{32x} - F_{34x} = m_3 a_{cmx} \]
\[ \Sigma F_y = -F_{32y} + F_{34y} = m_3 a_{cmy} \]
\[ \Sigma T = T_{32} + T_{34} + (R_{32x} x F_{32}) + (R_{34x} x F_{34}) = I_{cm} \alpha \]

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ R_{32y} & R_{32x} \end{bmatrix} \begin{bmatrix} F_{32x} \\ F_{32y} \\ T_{32} \end{bmatrix} = \begin{bmatrix} m_3 a_{cmx} - F_{32x} + F_{34x} \\ m_3 a_{cmy} + F_{32y} - F_{34y} \\ I_{cm} \alpha - (R_{32y} F_{32x} - R_{32x} F_{32y}) \end{bmatrix} \]

\[ F_{34} = \sqrt{F_{34x}^2 + F_{34y}^2} \quad F_{34} = 180 + \tan^{-1} \left( \frac{F_{34y}}{F_{34x}} \right) \]
\[ \sum F = ma \]
\[ \sum T = 1 \text{cm} \alpha \]
\[ \sum F_x = -F_{43x} + F_{45x} = m_4 \text{cm} x \]
\[ \sum F_y = F_{43y} - F_{45y} = m_4 \text{cm} y \]
\[ \sum T = T_{43} + (R_{43x} F_{43y}) + (R_{45x} F_{45y}) = 1 \text{cm} \alpha \]
\[ R_{43} \times F_{43} = R_{43x} F_{43y} - R_{43y} F_{43x} = \begin{vmatrix} R_{43x} & R_{43y} \\ F_{43y} & F_{43x} \end{vmatrix} \]
\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{43} & R_{43} & 1 \end{bmatrix} \begin{bmatrix} F_{43x} \\ F_{43y} \\ T_{43} \end{bmatrix} = \begin{bmatrix} m_4 \text{ cm} x - F_{43x} \\ m_4 \text{ cm} y - F_{43y} \\ 1 \text{ cm} \alpha - (R_{43x} F_{43y} - R_{43y} F_{43x}) \end{bmatrix} \]
\[ |F_{43}| = \sqrt{F_{43x}^2 + F_{43y}^2} \]
\[ F_{43} = \tan^{-1} \left( \frac{F_{43y}}{F_{43x}} \right) \]
\[ |F_{45}| = \sqrt{F_{45x}^2 + F_{45y}^2} \]
\[ F_{45} = \tan^{-1} \left( \frac{F_{45y}}{F_{45x}} \right) \]
26
Velocity

\[ \begin{align*}
  & \text{Velocity} \\
  & \text{vel}_x + i \text{vel}_y - m \text{vel}_z - \omega = 0 \\
  & \frac{\partial \text{vel}_x}{\partial t} + j \text{vel}_x \frac{\partial \text{vel}_y}{\partial t} - j \text{vel}_x \frac{\partial \text{vel}_z}{\partial t} = 0 \\
  & \frac{\partial \text{vel}_y}{\partial t} + j \text{vel}_y \frac{\partial \text{vel}_x}{\partial t} - j \text{vel}_y \frac{\partial \text{vel}_z}{\partial t} = 0 \\
  & \text{vel}_z \cos \theta_x + j \text{vel}_z \cos \theta_y = 0 \\
  & \text{vel}_z \sin \theta_x - j \text{vel}_z \sin \theta_y = 0 \\
  & \text{vel}_z = \frac{\text{vel}_z \sin (\theta_x - \theta_y)}{\sin (\theta_x - \theta_y)} \\
  & \text{vel}_x = \frac{\text{vel}_x \sin \theta_y}{\sin \theta_x} \\
  & \text{vel}_y = \text{vel}_y \\
  & \text{vel}_z = \text{vel}_z \cos \theta_x \\
  & \text{vel}_y = \text{vel}_z \cos \theta_y \\
  & \| \text{vel} \| = \sqrt{\text{vel}_x^2 + \text{vel}_y^2} \\
  & \theta = \tan^{-1} \left( \frac{\text{vel}_x}{\text{vel}_y} \right)
\end{align*} \]
Nucleation
\[ j \omega_2 e^{i \theta_1} + j \omega_3 e^{i \theta_2} - j \omega_2 e^{i \theta_3} = 0 \]
\[ j \frac{\partial e^{i \theta_2}}{\partial t} - j \omega_2 e^{i \theta_2} \frac{\partial e^{i \theta_2}}{\partial t} + j \frac{\partial e^{i \theta_3}}{\partial t} - j \omega_3 e^{i \theta_3} \frac{\partial e^{i \theta_3}}{\partial t} = 0 \]
\[ j \omega_2 e^{i \theta_2} - j \omega_2 e^{i \theta_2} + j \omega_3 e^{i \theta_3} - j \omega_3 e^{i \theta_3} + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ j \omega_2 e^{i \theta_2} - j \omega_2 e^{i \theta_2} + j \omega_3 e^{i \theta_3} - j \omega_3 e^{i \theta_3} + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ j \omega_2 (\cos \theta_2 + j \sin \theta_2) - j \omega_3 (\cos \theta_3 + j \sin \theta_3) + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ -j \omega_2 (\cos \theta_2 + j \sin \theta_2) - j \omega_3 (\cos \theta_3 + j \sin \theta_3) + m \mu_0^2 (\cos \theta_1 + j \sin \theta_1) = 0 \]
\[ j \omega_2 \cos \theta_2 - j \omega_2 \sin \theta_2 - j \omega_2 \sin \theta_2 - j \omega_2 \cos \theta_3 + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ j \omega_2 \sin \theta_2 - j \omega_2 \cos \theta_2 + j \omega_2 \cos \theta_3 - j \omega_2 \sin \theta_3 + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ j \omega_2 \sin \theta_2 - j \omega_2 \cos \theta_2 - j \omega_2 \cos \theta_3 - j \omega_2 \sin \theta_3 + j \mu_0 e^{i \theta_1} + m \mu_0^2 e^{i \theta_1} = 0 \]
\[ \alpha_1 = \alpha_2 = \frac{\mu_0 \cdot \mathbf{K}_1}{\mathbf{L}_2} = \frac{\mu_0 \cdot \mathbf{K}_2}{\mathbf{L}_2} \]
\[ K = g \sin \theta_1, E = 1 \cos \theta_1, \quad L = 1 \cos \theta_1, \quad \mu_0 \cos \theta_1 = 1 \cos \theta_1, \quad \mu_0 \sin \theta_1 = 1 \cos \theta_1 \]
\[ U = 1 \cos \theta_1, \quad V = 1 \cos \theta_1, \quad W = 1 \cos \theta_1 \]
\[ A_1 = \mu_0 (\cos \theta_1 + j \sin \theta_1) - m \mu_0^2 (\cos \theta_1 + j \sin \theta_1) \]